Speakers' Titles and Abstracts

Plenary Speakers

Laura DeMarco (Northwestern)

Complex dynamics and arithmetic equidistribution

I will explain a notion of arithmetic equidistribution that has found application in the study of complex dynamical systems. It was first introduced about 25 years ago, by Szpiro-Ullmo-Zhang, to analyze the geometry and arithmetic of abelian varieties. In this talk, I will relate these ideas to the Mandelbrot set, and I will present recent work that addresses dynamical questions and re-examines the unsolved problems about abelian varieties.

Irit Dinur (Weizmann Institute)

Unique games is ½-hard

The Unique Games Conjecture stands at the center of the area of hardness of approximation.

I will give some background and describe the status of this conjecture and recent progress towards proving it. This is based on work with Khot, Kindler, Minzer and Safra.

Martin Hairer (Imperial)

Bridging scales

One fascinating aspect of probability theory is the universal aspect of the objects it allows us to construct. The most well-known example of this phenomenon is the central limit theorem: for a very large class of collections of random variables, additive functionals that only depend weakly on any one element of the collection exhibit Gaussian behaviour in the limit. When taking time evolution into account, it turns out that in certain "cross-over regimes" the large-scale behaviour of a number of stochastic systems can formally be described by an ill-posed stochastic PDE.

Nalini Joshi (Sydney)

Symmetry through geometry

The search for symmetries of differential equations set in motion the development of extraordinarily important areas of mathematics. In this talk, we describe new perspectives on the corresponding problem for difference equations, by using simple, beautiful geometric structures revealed in our recent studies. The objects of this study are difference equations that arise as discrete versions of famous ODEs and PDEs. One of the many questions that arise is how to go from higher dimensional discrete systems to two dimensional ones. Another is how to construct such systems as walks on infinite lattices. I will show how the ideas of reflection groups and space-filling polytopes allow us to answer these questions.

Paul Seymour (Princeton)

Progress on some induced subgraph conjectures

A subgraph of another graph is 'induced' if it can be obtained just by deleting some vertices and their incident edges. There are several famous open questions about induced subgraphs, for instance:

(a) (Gyáfás-Sumner) If a graph has large (enough) chromatic number, and bounded clique number, then it contains any given tree as an induced subgraph.

(b) (Gyáfás) If a graph has large chromatic number and bounded clique number, then it contains a cycle of odd length at least 100 as an induced subgraph.

(c) (Erdös-Hajnal) For every graph H, every graph G not containing H as an induced subgraph has a clique or stable set of cardinality at least a (positive fractional) power of $|G|$.

We are still stuck on (a) and (c), but conjecture (b) is proved, and there is a new theorem which is a sort of mix of (a) and (c). We survey this and several other results.

This is joint work with Alex Scott, and partly with Maria Chudnovsky and Sophie Spirkl.

Marcelo Viana (IMPA)

Products of random matrices

A. Avila, A. Eskin and I are proving that the Lyapunov exponents of products of random matrices vary continuously with the underlying probability distribution. The 2-dimensional case had been proven in my student C. Bocker's thesis, in 2010. Much earlier, in his ICM 1983 address, R. Mañé had announced that the Lyapunov exponents of area-preserving diffeomorphisms can be killed by arbitrarily small perturbations (unless the diffeomorphisms happens to be uniformly hyperbolic). How to make sense of two such contrasting behavior of Lyapunov exponents?

Public Lecture

Julia Wolf (Bristol)

Using randomness to find structure

Somewhat counterintuitively, many phenomena involving randomness are actually extremely well understood. For example, when an unbiased coin is flipped a million times, we can predict the resulting number of heads with high accuracy.

Many other mathematical objects display random or at least random-like behaviour, which can be used to investigate their structural properties. This insight has been instrumental in, for instance, understanding century-old problems about patterns of prime numbers, analysing the structural properties of large networks, and designing efficient computer algorithms.

The talk will focus on these examples, and explore the limitations of this method as we understand them today.

Morning Speakers

Jonathan Bennett (Birmingham)

Induction on scales

Induction on scales is a method for proving functional inequalities (a simple example could be Hölder's inequality) by inducting on some auxiliary "regularity" parameter. This method comes in a variety of forms, and has been surprisingly effective across mathematics. The aim of this lecture is to illustrate the method in some elementary contexts, and describe one of many recent applications in harmonic analysis.

Clifford Cocks

The discovery of public key cryptography

Public key cryptography, which relies for its security on the difficulty of solving particular mathematical problems, is now ubiquitous and a critical component of online security. There were two parallel discoveries of this approach to cryptography during the 1970s, one in secret at Government Communications Headquarters in the UK, and the second, later discovery in academia in the USA. The talk will describe the discovery process, drawing out the similarities and differences between the two tracks.

David Conlon (Oxford)

How to build a hypergraph expander

We describe a variety of recent results whose shared theme is the effectiveness of applying the probabilistic method to algebraic structures in order to construct combinatorial objects with specified properties. In particular, we will discuss how to construct a family of sparse hypergraphs with strong expansion properties using Cayley graphs over elementary abelian 2-groups and the solution to a longstanding problem in extremal graph theory using random varieties over finite fields.

Charles Eaton (Manchester)

Classifying blocks of finite groups

Group algebras defined with respect to fields of prime characteristic are not in general semisimple, and so we approach the study of their modules in a different way to those defined with respect to fields of characteristic zero. The stucture of a module, e.g., its submodules and the relationship between them, becomes more important. Morita equivalence of algebras captures the structure of modules, and we are interested in classifying Morita equivalence classes of algebras, in particular blocks of finite groups (which are indecomposable subalgebras of the group algebra). Central to all of this is Donovan's conjecture, which predicts that there are only finitely many Morita equivalence classes of blocks of each defect (an invariant associated to a block).

I will give a survey of the problem of identifying Morita equivalence classes of blocks of finite groups, including progress on Donovan's conjecture.

Mike Fellows (Bergen)

The multivariate revolution in algorithmics

Until recently, the study of algorithms and complexity was dominated by the one dimensional framework of P versus NP. In the last few years, there has been an explosion of interest in the multivariate perspective on the subject provided by parameterized complexity. The talk will exposit the basic ideas of the multivariate approach, where in addition to the basic measurement n of the number of bits in a legal input to a decision problem, one also has an associated parameter k, which may be a vector of secondary structural aspects of the problem instance input. Parameterized complexity was originally motivated by the deep results of Robertson and Seymour in their graph minors project. The FPT result that they proved, where the input is a pair (G,H) where G is a graph on n vertices, and the smaller graph H is considered the parameter, is based on deep combinatorial structure theory. The talk will explain how parameterized complexity is fundamentally about the possibility of parameterized structure theory that can be exploited in designing algorithms, and via the kernelization characterization of FPT, deeply connected to extremal combinatorial theory. The talk will also survey some of the current frontiers and challenging open problems of the subject.

Victoria Gould (York)

Biordered sets of idempotents

Idempotents are ubiquitous in mathematics, representing functions where repeated powers yield the same result. Therefore, the mathematically most natural – and general – context within which to study them is that provided by semigroup theory. Indicators of this pervasive viewpoint include (i) the work on the idempotents of Stone–Čech compactifications and their applications to combinatorics, (ii) idempotent tropical matrices and connections to the geometry of tropical polytopes, (iii) the fundamental role played by idempotents in Putcha–Renner theory of reductive algebraic monoids, as well as (iv) their importance in the representation theory of algebras, groups and semigroups.

Every semigroup embeds into one generated by idempotents, and many semigroups of transformations are idempotent generated. Thus, idempotent generated semigroups are both universal and natural objects. For a semigroup S the set $E = E(S)$ of idempotents of S comes equipped with the structure of a biordered set and thus possesses simultaneously algebraic and order-theoretic properties. For any biordered set E there is a freeest semigroup generated by E having E as its biordered set of idempotents, the free idempotent generated semigroup FIG(E). Any idempotent generated semigroup S with biordered set $E = E(S)$ is then a natural homomorphic image of FIG(E). The exact relationship between FIG(E) and S is intricate and has led to the development of extensive methodology.

Without presuming any specialist knowledge I hope to convince the audience that the theory of biordered sets and free idempotent generated semigroups is deep and rich. The talk will have two focuses. The first is to show that biordered sets are sophisticated enough to capture, via semigroups of the form FIG(E), the structure of any group. The second is related to a decision problem. If E is finite, then FIG(E) is finitely presented. In this case, Dolinka, Yang and I have shown that the word problem for FIG(E) is equivalent to a family of constraint satisfaction problems involving rational subsets of direct products of pairs of maximal subgroups of FIG(E). As an application, we obtain decidability of the word problem for an important class of examples. Moreover, our techniques allow us to prove that for finite E the semigroup FIG(E) although not (von Neumann) regular, always possesses a similar but weaker property.

Daniel Král' (University of Warwick)
Analytic representations of large discrete structures

The theory of combinatorial limits provides analytic tools to study large discrete structures. Such tools have found applications in various areas of computer science and mathematics; they are also closely linked to the flag algebra method, which changed the landscape of extremal combinatorics. We will present an introduction to this rapidly developing area of combinatorics and survey some of the recent results, in particular those concerning graphs and permutations.

Holly Krieger (Cambridge)

Unlikely intersections in complex dynamics

I will discuss the arithmetic-geometric notion of unlikely intersections and its applications in the setting of moduli spaces of rational maps of the Riemann sphere, focusing on the dynamical André-Oort question. First studied by Baker-DeMarco, a product version of the dynamical version of this question was answered by Ghioca, Nguyen, Ye, and myself. I will discuss our work, which relies on arithmetic equidistribution of points of small height as well as the combinatorics of external rays of the Mandelbrot set. Time permitting, I'll discuss another recent result of this type (joint with DeMarco and Ye) concerning common preperiodic points of two dynamical systems.

Marta Mazzocco (Loughborough)

The geometry behind the q-Askey scheme

The Askey–Wilson polynomials are a special case of Macdonald polynomials that are basic hypergeometric polynomials, i.e. polynomials expressed in terms of the q-Hypergeometric series (the q-difference version of the classical hypergeometric series). The basic hypergeometric polynomials are organised into a hierarchy called q-Askey scheme, in which different families of polynomials are connected by directed arrows, each arrow representing a degeneration in which the parameters and the variable (or in come cases the exponentiated parameters and variables) are rescaled by some appropriate powers of ε and then the limit $\varepsilon \to 0$ is taken. In this talk the speaker will give a new approach to study degenerations as well as dualities of basic orthogonal polynomials based on the fact that all families of polynomials in the q-Askey scheme admit a certain algebra of symmetries (the Zhedanov algebra and its degenerations) and in the limit as $q \to 0$ this algebra becomes the coordinate ring of a certain (decorated) character variety.

Ian Morris (Surrey)

Towards the construction of high-dimensional measures on self-affine sets

Many familiar examples of fractals such as the Sierpiński gasket, the von Koch curve and hyperbolic Julia sets have the property of containing multiple smaller copies of themselves, which are either exact copies of the original set (in the case of the Sierpinski gasket and von Koch curve) or conformally distorted copies (in the case of Julia sets). When the smaller copies are not subject to any conformality constraints the resulting fractal becomes substantially more difficult to analyse. Self-affine sets are characterised by the property of being equal to a union of linearly distorted smaller copies of themselves and have been a source of challenging open problems since they were first investigated systematically in the 1980s. In this talk I will describe some recent progress on the construction of high-dimensional measures on self-affine sets, focusing on my recent work with Pablo Shmerkin, Antti Käenmäki and Jairo Bochi.

Simon Smith (Lincoln)

Infinite permutation groups

In this talk I will summarise recent developments in the theory of infinite permutation groups. My talk will primarily concern infinite permutation groups that are *subdegree-finite*: in such groups all orbits of point stabilisers are finite. For example, the group Aut($\mathbf{Q}$, <) of order-preserving permutations of the rational numbers is not subdegree-finite, whereas the group Aut(T_3) of isometries of the 3-valent regular tree T_3 (in its action on the vertices of T_3) is subdegree-finite.

Subdegree-finite permutation groups arise as the natural permutation representations of totally disconnected and locally compact groups (tdlc groups), and these permutation representations have underpinned the proofs of a number of important results that illuminate the structure of compactly generated tdlc groups.

When examining permutation groups, one typically looks first at those that are primitive. Primitive permutation groups are indecomposable in some sense, and are often thought of as being the "atoms" of permutation group theory. My talk will conclude with a recent result describing the structure of subdegree-finite primitive permutation groups.

Hendrik Weber (Warwick)

The stochastic quantisation equation – scaling limits, meta-stability and the role of infinity

This talk is concerned with stochastic partial differential equations (SPDEs) driven by a singular noise term, namely space-time white noise. These equations naturally appear in diverse contexts, but their rigorous mathematical treatment is challenging, because solutions are in general very irregular. They often have to be interpreted as Schwartz distributions rather than as functions, and equations often have to be interpreted in a "renormalized" sense, i.e. some formally infinite terms have to be removed.

I will focus on a particular SPDE, namely the stochastic quantisation equation. This equation was first proposed by Parisi and Wu in the early 1980s in the context of constructive field theory, but I will discuss mainly two very different situations where the equation arises naturally: as a scaling limit for an interacting particle system and as a model for meta-stability. I will explain in particular, which role the "infinite terms" play in each of these situations.

Workshops

Algebra

Organiser: Ben Martin (Aberdeen)

Gavin Brown (Warwick)

Flops and Calabi-Yau potentials

I will describe an on-going project with Michael Wemyss (Glasgow) in which we construct smooth 3-fold flops from the ingredients of Donovan-Wemyss contraction algebras, which are certain finite-dimensional non-commutative algebras related to the flopping geometry, and their Calabi-Yau potentials. Our methods are enough to construct and distinguish flops that conventional invariants cannot. I will explain the geometry in detail, with a brief historical comparison via an ADE type classification, and show how it relates to the non-commutative algebra.

Tim Burness (Bristol)

The length and depth of a group

The length of a finite group G is defined to be the maximal length of an unrefinable chain of subgroups from G to 1; this notion has been the subject of numerous papers dating back to the 1960s, especially in the context of simple groups. In recent joint work with Martin Liebeck and Aner Shalev, we study a related concept, which we call the depth of G. This is the minimal length of an unrefinable chain of subgroups from G to 1 and it is interesting to compare these two parameters. In this talk, I will focus on the depth of simple groups. In particular, I will discuss our classification of the simple groups of minimal depth, and I will explain the somewhat surprising fact that alternating groups have bounded depth. I will conclude by highlighting some results for arbitrary finite groups and I will briefly outline some recent work on an appropriate notion of length and depth for algebraic groups.

Laura Ciobanu (Heriot-Watt)

Conjugacy growth in groups

In this talk I will discuss asymptotic and formal aspects of conjugacy growth in several classes of infinite groups, such as hyperbolic, right-angled Artin, or wreath products.

In particular, I will give quantitative results that describe the relation between standard and conjugacy growth.

Maud De Visscher (City University)

Kronecker coefficients and partition algebras

A fundamental problem in the representation theory of the symmetric group is to describe the coefficients in the decomposition of a tensor product of two simple modules. These coefficients are known as the Kronecker coefficients and apart from a few small families of cases, there is no combinatorial formula to describe them.

In joint work with Christopher Bowman and Rosa Orellana, we have reinterpreted the Kronecker coefficients in the setting of the partition algebra. In this talk I will explain the connection between the symmetric group and the partition algebra and also present some new results on Kronecker coefficients using this approach (joint work with Christopher Bowman and John Enyang).

James East (Western Sydney)

Congruences on diagram monoids

A congruence on a semigroup is an equivalence relation that is compatible with the semigroup operation. Congruences play a role in semigroup theory akin to that of normal subgroups in group theory; they govern the formation of quotient semigroups, are kernels of semigroup homomorphisms, and so on. In a major 1952 paper, A.I. Mal'cev classified the congruences of a full transformation semigroup: i.e., a semigroup consisting of all self-maps of a fixed set. In the finite case, the lattice of all such congruences forms a chain. In the infinite case, the situation is far more complicated, but Mal'cev gives a succinct description nevertheless. This talk will report on some recent work in which we classify the congruences on certain diagram monoids; these include the partition, Brauer and Temperley-Lieb monoids, for example. The finite case is joint work with James Mitchell, Nik Ruskuc and Michael Torpey (all at St Andrews), and the infinite is joint with Nik Ruskuc.

Brent Everitt (York)

(Co)homology of arrangements

An arrangement is a finite collection of linear hyperplanes in some vector space. This talk is a survey of answers to the question, "what is the cohomology of an arrangement?".

Valentina Grazian (Aberdeen)

The classification of simple fusion systems

In finite group theory, the word fusion refers to the study of conjugacy maps between subgroups of a group. The modern way to solve problems involving fusion is via the theory of fusion systems. A saturated fusion system on a p-group S is a category whose objects are the subgroups of S and whose morphisms are the monomorphisms between subgroups which satisfy certain axioms motivated by conjugacy relations. The aim of this talk is to present the state of the art in the classification of simple fusion systems.

Marianne Johnson (Manchester)

Two-letter identities of the bicyclic monoid

Fix a word over a two letter alphabet. Which words have the property that any substitution of the letters for elements of the bicyclic monoid yields equality between the corresponding two products? I will explain how this question can be answered by considering certain lattice polytopes derived from the word. This is based on joint work with Ngoc Mai Tran.

Analysis and Probability

Organisers: Tadahiro Oh and Jim Wright (Edinburgh)

Tony Carbery (Edinburgh)

A multilinear Maurey factorisation theorem

A famous theorem of B. Maurey states that if T is a positive linear map from a normed lattice Y to the Borel measurable functions on a locally finite Hausdorff topological space X which is bounded from Y into $L^q(X)$ with $0 < q < 1$, then T factorises through L^1, that is, there exists a suitable probability density w such that T is bounded from Y into $L^1(w^{(q-1)/q})$. This result and its friends have many well-known applications in harmonic analysis. In joint work with Stefan Valdimarsson we give a multilinear analogue of this theorem.

Justin Forlano (Heriot-Watt)

Almost sure global well-posedness for the BBM equation with infinite L^2 initial data

In this talk, we discuss the Cauchy problem for the Benjamin-Bona-Mahony equation (BBM) posed on the one-dimensional torus. With respect to random initial data of strictly negative Sobolev regularity, we prove that BBM is almost surely globally well-posed. The argument employs a second order expansion in terms of the random initial data, with the *I*-method used to obtain an a priori bound on the growth of the 'residual' part.

Máteé Gerencsér (IST Austria)

Quasilinear singular SPDEs within regularity structures

We present an approach for quasilinear singular SPDEs that enables one to fit them into the theory of regularity structures. This way many general results available can be leveraged, and in particular a solution theory can be obtained for an essentially optimal class of equations. Joint work with Martin Hairer.

Xue-Mei Li (Imperial)

Conservation law, stochastic averaging, and intrinsic geometry

The study of a singularly perturbed stochastic dynamics is concerned with extracting a meaningful limit from a family of diffusion operators with a parameter which has no classical meaning when the parameter is taken to zero. We will describe a geometric method for reducing singular perturbation perturbation problem with symmetries to slow-fast dynamics, and methods for solving the latter.

Diogo Oliveira e Silva (Birmingham)

Sharp Strichartz inequalities for fractional and higher order Schrödinger equations

It has long been understood that Strichartz estimates for the homogeneous Schrödinger equation correspond to adjoint Fourier restriction estimates on the paraboloid. The study of extremizers and sharp constants for the corresponding inequalities has a short but rich history. In this talk, I will summarize it briefly, and then specialize to the case of certain planar power curves. A geometric comparison principle for convolution measures can be used to establish the corresponding sharp Strichartz inequality, and to decide whether extremizers exist. The mechanism underlying the possible lack of compactness is explained by the behaviour of extremizing sequences and will be described via concentration-compactness. Time permitting, I will show how this resolves a dichotomy from the recent literature concerning the existence of extremizers for the fourth order Schrödinger equation in one spatial dimension. This talk is based on joint work with Gianmarco Brocchi and René Quilodrán.

Oana Pocovnicu (Heriot-Watt)

Long time regularity of the 2D Euler-Poisson system for electrons with vorticity

The Euler-Poisson system for electrons is one of the simplest two-fluid models used to describe the dynamics of a plasma. From the point of view of analysis, it can be re-written as a quasilinear hyperbolic PDE. In this talk, we will discuss the long time existence for the two-dimensional Euler-Poisson system, with a particular attention to the dependence of the time of existence on the size of the vorticity. This talk is based on joint work with Alexandru Ionescu (Princeton).

Vedran Sohinger (Warwick)

Gibbs measures of nonlinear Schrödinger equations as limits of many-body quantum states in dimension $d \leq 3$

Gibbs measures of nonlinear Schrödinger equations are a fundamental object used to study low-regularity solutions with random initial data. In the dispersive PDE community, this point of view was pioneered by Bourgain in the 1990s. We prove that Gibbs measures of nonlinear Schrödinger equations arise as high-temperature limits of appropriately modified thermal states in many-body quantum mechanics. We consider bounded defocusing interaction potentials and work either on the d-dimensional torus or on $\mathbf{R}^d$ with a confining potential. The analogous problem for $d = 1$ and in higher dimensions with smooth non translation-invariant interactions was previously studied by Lewin, Nam, and Rougerie by means of entropy methods. In our work, we apply a perturbative expansion of the interaction, motivated by ideas from field theory. The terms of the expansion are analyzed using a diagrammatic representation and their sum is controlled using Borel resummation techniques. When $d = 2, 3$, we apply a Wick ordering renormalisation procedure. Moreover, in the one-dimensional setting our methods allow us to obtain a microscopic derivation of time-dependent correlation functions for the cubic nonlinear Schrödinger equation. This is joint work with Juerg Froehlich, Antti Knowles, and Benjamin Schlein.

Pavlos Tsatsoulis (Warwick)

On the long time behavior of the dynamic Φ_2^4

We consider the dynamic Φ_2^4 model (or stochastic Allen–Cahn equation) formally given by the SPDE

$$(\partial_t - \Delta)\phi = -\phi^3 + \phi + \sqrt{2\varepsilon}\,\xi \qquad (\Phi_2^4)$$

where ξ is a space-time white noise in 2+1 dimensions and $\varepsilon > 0$.

The above equation is a 'toy example' in a wider class of singular SPDEs, i.e. SPDEs where classical solution theories do not apply. This is basically due to the irregularity of ξ which forces the solution to be a distribution and not a function. Hence the term ϕ^3 in (Φ_2^4) is not well-defined.

Although at first glance the singularity of the system seems a major obstacle, a local solution theory can be obtained by canceling out the divergences of the non-linear cubic term through a 'Wick renormalisation'.

In this talk we will give a short introduction on the topic and review results beyond local solution theory related to the long time behavior of (Φ_2^4).

Combinatorics

Organiser: Sergey Kitaev (Strathclyde)

Jess Enright (Edinburgh)

Changing times in temporal graphs to limit disease spread

Temporal graphs (in which edges are active only at specified time steps) are an increasingly important and popular model for a wide variety of natural and social phenomena. We're interested in graph modification in a temporal setting.

Motivated by cows and rumours, I'll talk about a particular modification problem in which we assign times to edges so as to maximise or minimise reachability sets within a temporal graph. I'll mention an assortment of complexity results on these problems, showing that they are hard under a wide variety of restrictions. In particular, if edges can be grouped into classes that must be assigned the same time, then the problem is hard even on directed acyclic graphs when both the reachability target and the classes of edges are of constant size, as well as on an extremely restrictive class of trees. At least one version of this problem is W[1]-hard when parameterised by the vertex cover number of the instance graph. The situation is slightly better if each edge is active at a unique timestep - in some very restricted cases the problem is solvable in polynomial time. (This is joint work with Kitty Meeks)

Ji-Hwan Jung (Sungkyunkwan, Korea)

Oriented Riordan graphs

An oriented graph is a directed graph having no symmetric pair of directed edges. Let G^σ be a simple graph with an orientation σ, which assigns to each edge a direction so that G^σ becomes a directed graph. With respect to a labeling, the *skew-adjacency matrix* $\mathcal{S}(G^\sigma)$ is the (-1,0,1)-skew symmetric matrix $[s_{ij}]_{\{1\leq i,j\leq n\}}$ where s_{ij} = 1 and s_{ij} = -1 if $i \rightarrow j$ is an arc of G^σ, otherwise $s_{ij}=s_{ji}=0$.

In this talk, we use the theory of Riordan matrices to define the notion of an *oriented Riordan graph*. Then we introduce their structural properties including a fractal property, a decomposition theorem and so on. This is joint work with Gi-Sang Cheon.

Kitty Meeks (Glasgow)

Exploiting structure in multi-layer networks: a case study on motif counting

Many real-world systems are most naturally modelled by "multi-layer" networks, which allow for different types of connections between entities; it is therefore important to develop efficient algorithms to extract information from such networks. However, most existing results concerning the structural properties of graphs/networks which allow us to solve NP-hard problems efficiently consider only the case of a "single-layer" graph (in which each pair of vertices is either adjacent or not). A natural question to ask is whether, if each individual layer has well-understood structure which allows the design of efficient algorithms, we can still exploit this structure to solve problems on the combined, multi-layer network. We address this question for the specific problem of counting small substructures in the network: in most cases the problem becomes intractable on the combined network, but we identify one case in which structural restrictions on the individual layers can be exploited effectively. This is joint work with Jessica Enright (University of Edinburgh).

Arseny Shur (Ural Federal University)

Subword complexity and power avoidance

Abstract: Two very important numerical characteristics of a finitary infinite sequence (word) are its subword complexity and critical exponent. The first of them is the function returning the number of distinct segments of given length in the word; the second characteristic is the supremum of orders of all repetitions in this word. These characteristics are related: for example, for the words of subword complexity $n+1$ (Sturmian words) the critical exponent is at least $2 + \varphi$, where φ is the golden ratio. However, this relation can be quite complicated.

In this talk, we overview recent results for the following setting: a class of sequences is specified by fixing an alphabet and bounding the allowed critical exponents from above; what is the minimum / minimal / maximum / maximal subword complexity in this class? The key roles are played by both the classical sequences, like the Thue-Morse word and the Fibonacci word, and some words which appear recently, like the twisted Thue-Morse word and the 1-2-bonacci word.

Jason Smith (Strathclyde)

The Poset of Graphs

We introduce the poset of all unlabelled graphs, up to isomorphism, with the partial order $H < G$ if H is an induced subgraph of G. We will present some results on the Möbius function and topology of this poset. In particular we will discuss a class of graphs for which the Möbius function equals the Catalan and Fibonacci numbers.

Katherine Staden (Oxford)

Stability via symmetrisation

The method of symmetrisation was employed by Zykov in 1949 to give a new proof of Turán's theorem in graph theory. Since then it has been useful in other extremal problems. In this talk, I will discuss a sufficient condition for the stability property of extremal graph problems that can be solved via this method. Our criterion is stated in terms of the analytic limit version of the problem. We show that, for example, it applies to the inducibility problem for an arbitrary complete bipartite graph B, which asks for the maximum number of induced copies of B in an n-vertex graph. Joint work with Hong Liu, Oleg Pikhurko and Maryam Sharifzadeh.

Arthur L.B. Yang (Nankai University)

Compatibility of generalized Eulerian polynomials

In the study of the real-rootedness of the independence polynomials of clawfree graphs, Chudnovsky and Seymour introduced the method of compatible polynomials, which turned out to be a very powerful tool for the study of real-rootedness. In this talk, I will talk about recent progress of some conjectures on the real-rootedness of various Eulerian polynomials. Special emphasis will be given to Brenti's type D Eulerian polynomials and Dilks-Petersen-Stembridge's type D affine Eulerian polynomials. The last part of the talk will provide an alternative approach to the real-rootedness of Eulerian polynomials via the Hermite-Biehler theorem.

Viktor Zamaraev (Durham)

On the factorial classes of bipartite graphs

This talk is devoted to the problem of characterizing the family of factorial classes of graphs, i.e., hereditary classes in which the number of n-vertex graphs grows as a factorial type function. First, we will discuss the background and motivation of the problem. Then we will consider a special case of characterizing the family of factorial classes of bipartite graphs. We will present recent results in this direction and discuss some open problems.

Dynamics

Organiser: Jonathan Fraser (St Andrews)

Jonas Azzam (Edinburgh)

Wasserstein distance and rectifiability of measures

A curve of finite length has tangents at almost every point (with respect to arclength), which follows from arclength parametrizing the curve and applying Rademacher's theorem. Thus, as we zoom in at almost every point on the curve, the curve is getting flatter and flatter. However, asymptotic flatness at every point is not sufficient for a curve to be rectifiable. A result of Bishop and Jones, however, shows that a necessary and sufficient condition for rectifiability is that how straight the curve is in a ball of radius r is square integrable over the radii. This is analogous to results in harmonic analysis that classify the pointwise differentiability of a function in terms of a Dini condition. In this talk, we will discuss a variant of this result for Radon measures in Euclidean space and show that, for pointwise doubling measures, rectifiability of a measure is equivalent to a similar Dini condition that measures 'flatness' in terms of how close (using a variant of Wasserstein distance) the measure is to resembling planar measure. This is joint work with Tatiana Toro and Xavier Tolsa.

Jairo Bochi (PUC de Chile, Santiago)

Optimization of Lyapunov exponents

I'll explain some basic results about optimization (e.g., maximization) of Lyapunov exponents of products of matrices driven by a dynamical system, comparing them with corresponding results on optimization of ergodic averages.

Xiong Jin (Manchester)

Dimension of projections of Gibbs measures on self-conformal sets

We extend the seminal work of Hochman and Shmerkin on orthogonal projections of self-similar measures to Gibbs measures on self-conformal sets (in the plane). The proofs rely on symbolic dynamics and compact group extension theorem. We also formulate an analogue of the 'dense rotation' condition in the conformal iterated function system and provide an easy-to-check necessary condition for it to hold for Julia sets defined by the polynomials $z^2 + c$. This is joint work with Catherine Bruce.

Natalia Jurga (Warwick)

A dimension gap for Bernoulli measures for the Gauss map

It is well known that the Gauss map T: $[0,1] \setminus \mathbf{Q} \to T$: $[0,1] \setminus \mathbf{Q}$ given by $T(x)= 1/x \pmod 1$ has an absolutely continuous invariant probability measure μ_T given by

$$\mu_T(A) = (1/\log 2) \int_A 1/(1+x)\mathrm{d}x.$$

Let $\mu_{\mathbf{p}}$ denote the Bernoulli measure associated to the countable probability vector **p**, projected to [0,1] in the usual way. Kifer, Peres and Weiss showed that the Bernoulli measures for the Gauss map satisfy a *dimension gap* meaning that there exists $c > 0$ such that

$$\sup_{\mathbf{p}} \dim_H \mu_{\mathbf{p}} < 1- c. \qquad (1)$$

Moreover, they showed that $c \geq 10^{-7}$. Their proof was based on considering sets of large deviations for the asymptotic frequency of certain digits from the one prescribed by μ_T.
In this talk we will discuss an alternative proof of (1) which instead reduces to obtaining good lower bounds on the asymptotic variance of a class of potentials.

Tom Kempton (Manchester)

On the Hausdorff dimension of Bernoulli convolutions

Bernoulli convolutions are a simple family of self-similar measures with overlaps. The problem of determining which parameters give rise to Bernoulli convolutions of dimension one has been studied since the 1930s, and is still far from being completely solved. For algebraic parameters, we show how to give an expression for the dimension of the Bernoulli convolution in terms of products of matrices. This allows us to conclude that the Bernoulli convolution has dimension one in many examples where the dimension was previously unknown. This is joint work with Shigeki Akiyama, De-Jun Feng and Tomas Persson.

Phil Rippon (Open University)

Slow escaping points of transcendental entire functions

Let f be a transcendental entire function. Some years ago we showed that for any such function f there always exist points which escape to ∞ arbitrarily slowly. The proof depends on two results which together show that the image of any suitably large annulus under f must cover another suitably large annulus.

We discuss these covering results and show to what extent the slow escaping result can be adapted to give arbitrarily slow escaping orbits that remain in the same component of $\{z: |f(z)| > 1\}$, a so-called *tract* of f. This is joint work with Gwyneth Stallard.

James Robinson (Warwick)

Minimal periods in Lipschitz ODEs

Suppose that we have a Lipschitz continuous differential equation on a Banach space X: $\mathrm{d}x/\mathrm{d}t = f(x)$, where $|f(x) - f(y)| \leq L\,|x - y|$. Using a geometric argument, Yorke showed that if X is $\mathbf{R}^n$ equipped with the usual Euclidean norm then any non-constant periodic orbit must have period at least $2\pi/L$. Busenberg, Fisher, and Martelli gave an analytical proof of the same result valid in any Hilbert space, and showed that in an arbitrary Banach space the minimal period is at least $6/L$. I will give proofs of these results, show that the minimal period is strictly larger than $6/L$ in any strictly convex Banach space (e.g. in $\mathbf{R}^n$ with the ℓ^p norm, $1 < p < \infty$), and discuss some related open problems. This is in part joint work with Michaela Nieuwenhuis and Stefan Steinerberger.

Gwyneth Stallard (Open University)

Fast escaping points of transcendental entire functions

This talk concerns complex dynamics, in particular, the iteration of analytic functions of the complex plane with an essential singularity at infinity. Much of the recent research in this area has been motivated by Eremenko's conjecture that the escaping set (the set of points which escape to infinity under iteration) has no bounded components. The strongest general results on this conjecture have been obtained by studying the fast escaping set which consists of those points which escape to infinity 'as fast as possible'. We discuss how this has led to new insights into possible structures of the escaping set such as 'infinite spiders' webs' and 'Cantor bouquets of curves'. This is joint work with Phil Rippon.

History of Mathematics

Organiser: Isobel Falconer (St Andrews)

Rosemary Bailey (St Andrews)

Latin squares: Some history, with an emphasis on their use in designed experiments

In the 1920s, R. A. Fisher, at Rothamsted Experimental Station in Harpenden, recommended Latin squares for agricultural crop experiments. At about the same time, Jerzy Neyman developed the same idea during his doctoral study at the University of Warsaw.

However, there is evidence of their much earlier use in experiments.

Euler had made his famous conjecture about Graeco-Latin squares in 1782. There was a spectacular refutation in 1960.

I shall say something about the different uses of Latin squares in designed experiments. This needs methods of construction, of counting, and of randomization.

Fisher and Neyman had a famous falling out over Latin squares in 1935 when Neyman proved that use of Latin squares in experiments gives biased results. A six-week international workshop in Boulder, Colorado in 1957 resolved this, but the misunderstanding surfaced again in a Statistics paper published in 2017.

Tony Mann (Greenwich)

Mathematics instructors at the Royal Naval College, Greenwich

The Royal Naval College, which opened at Greenwich in 1875, employed a very distinguished sequence of Professors of Mathematics, including William Burnside and Louis Milne-Thompson. This talk will investigate rather some of the mathematical instructors who supported the Professors, including John Knox Laughton, who later achieved eminence as a naval historian, and Richard Wormell, a future President of the Association for the Improvement of Geometrical Teaching.

Ursula Martin (Oxford)

The rise of modern patronage, the social, and the impact of mathematics

The mid-twentieth century saw the humanistic approach of mathematics as an intellectual endeavour pursued for its own sake, increasingly challenged by an instrumental view of the discipline as contributing to the nation. State funders, seeking more than Vannevar Bush's forceful arguments that unfettered intellectual curiosity gives rise to unpredictable and useful discoveries, now look for evidence of return on investment. Such evidence is collected in the UK in a standard format, allowing analysis of how research in mathematics gives rise to impact, going beyond more usual patents or start-up companies, to capture impacts on policy, and influence, and in turn, inform new models of government and corporate patronage.

Close analysis of these shows that the mathematics community have been right all along in challenging narrow instrumental views, and that the impact of mathematics is brought about largely through long term relationships, interdisciplinarity and the skills of mathematicians themselves in exploring new problems and finding new collaborators. Peversely, the richer such an ecosystem of mathematics and users of mathematics, the easier it is to find apparent linear chains leading to impact, but the less representative these become.

Amirouche Moktefi (Tallinn University of Technology)

Playing by the rules: Venn versus Carroll

The role of diagrams in mathematical proofs is disputed. Yet diagrams have long been used in various mathematical disciplines. In logic, they knew a golden age after their popularisation by Leonhard Euler in his Letters to a German Princess (1768). By the end of the nineteenth century, several schemes were in existence, and to some extent, in competition. In this talk, we will expose and compare two diagrammatic methods introduced by British logicians John Venn and Lewis Carroll. Both were invented to handle logic problems known as elimination problems which consist in finding what information regarding any combination of terms follows from a set of premises. For the purpose, Venn published in 1880 a scheme offered as an improvement over Euler's well-known circles. The method consisted in representing the complete information contained in the premises on a single diagram, then to see 'at a glance' the conclusion regarding specific terms. An inconvenience of this scheme, as pointed out by Louis Couturat (1914), is that it does not really tell how the conclusion is to be 'extracted' from the diagram. A rival scheme, published in 1886 by Lewis Carroll, demands that information is transferred from the premises-diagram to another diagram that would depict the conclusion. This transfer is achieved by following rules which are explicitly defined and strictly applied. Although both Venn and Carroll introduced diagrammatic methods for the problem of elimination, they differ in their practices and demands on how a diagram ought to be manipulated. Venn appealed to imagination to work out the conclusion with a single diagram while Carroll applied rules on a diagram to derive other diagrams. The former method was said to lack rigor, but the latter may be accused of lacking naturalness and economy. This difference of practices, and the philosophical views that they embody, will be shown to resurface in the recent debates on the role of diagrams in mathematical practice.

Edmund Robertson (St Andrews)

Mary Everest Boole: the first mathematical psychologist

Mary Everest Boole was only 32 years old in 1864 when her husband, the mathematician George Boole, died leaving her with five daughters to bring up. In this talk we will look at Mary Boole's life and her contributions to mathematics teaching. She published many books on teaching in general and teaching mathematics in particular, for example Lectures on the Logic of Arithmetic (1903) and

Philosophy and Fun of Algebra (1909). We give quotes from her books to illustrate her ideas which, although over 100 years old, seem surprisingly relevant today.

Brigitte Stenhouse (Open University)

Embracing Nature in Formulae: The Hidden Mathematics of Mary Somerville

In the 1830s, after the publication of her first book Mechanism of the Heavens, Mary Somerville was known throughout the UK and continental Europe as an expert in analysis, and its applications to astronomy. Her next work centred on the theme of 'the physical sciences', which she claimed were united by the "bond of analysis" which would "ultimately embrace almost every subject in nature in its formulae".[1] However, these formulae are conspicuously missing from both this and all her future publications.

An unusual situation is made ever more peculiar by the existence of two unpublished manuscripts Somerville completed in 1834, both of which are explicitly mathematical and would have slotted in perfectly to her analytical agenda. I will outline and contextualise the content of one of these papers, and open the question of why mathematics vanished from Somerville's published works.

1. Somerville, M. *On the Connexion of the Physical Sciences*. John Murray, Albemarle Street, London 1834.

Kevin Tracey (Swansea University and the Science Museum, London)

Calculating Value: Reading the Scribal Technologies of Early Modern Mathematics

Comprised of some 3,300 volumes printed between 1486 and 1800, the Rare Books Collection of the Science Museum, London, contains a significant number of texts on astronomy, geometry, arithmetic, physics, and natural philosophy as printed and read in the early modern period. As such, the collection is a remarkable locus in which to explore how the contemporary readers of such texts used and understood their mathematical materials. Altered and adorned by signatures, annotations, corrections, and disputations, these material artefacts display evidence of users wrestling with the key mathematical concepts of their day at sites of practice including universities, shipyards, and marketplaces alike.

Drawing upon three uniquely-annotated texts, this talk situates early modern mathematics and its readers in their appropriate historical, methodological and philosophical contexts. It will move from a multi-edition sammelband used at the University of Wittenberg in the late sixteenth century to the European roots of the volvelles and paper instruments as presented in Thomas Blundeville's popular Exercises (1594). A detailed presentation of the use and preservation of John Seller's Pocket Book (1677) will then demonstrate how the transmission and reception of trigonometry and spherical astronomy were aided by early modern reading practices well into the eighteenth century.

Presenting the 'scribal technologies' utilised by early modern individuals, this paper seeks to shed further light on the intellectual methods such readers applied to their personal mathematical practice. Mathematics existed in the period as part of a wider continuum of print: one including instruments, maps, and globes alongside published texts. How such users 'read' their materials is therefore of significant value to our understanding of their intellectual and material interests, and furthermore, to the history of mathematics, the history of science, and the history of the book.

Jane Wess (Edinburgh)

Maths and Maps: A comparison of Mathematical Theory and Cartographic Practice

The talk is a work in progress focusing on the long 19th century in Britain. It will look at three aspects of maps: projections, the representation of heights, and the four-colour theory. In each of the three cases the talk will ask what the theoretical situation was, what was happening at the Royal Geographical Society in London in terms of what was published, discussed and acquired, and what has been done to compare theory and practice more generally.

It will argue that there was little or no engagement with mathematical theory in the 19th century with respect to projections. While the mathematical theory of projections was sophisticated by this time, map makers tended to adhere to tradition. A survey of projections in the collection of the Royal Geographical Society demonstrates the almost exclusive prevalence of Mercator’s projection. Map makers were slow to adopt contour lines, even though isolines representing magnetic and barometric phenomena were popular. Map makers were either not aware of the four colour theory or did not consider it useful. Since studies of maps in the mid to late twentieth century the web has made it very much easier to undertake large scale surveys, but so far the situation is far from clear. Did the map makers not know or did they not care?

This paper forms part of an investigation into ‘The Unreasonable Effectiveness of Mathematics’ concept, first proposed in 1960 by Eugene Wigner. In the case of longitude it was found to be not applicable, in that the mathematics was so difficult it was beyond the abilities of the vast majority, so contributed nothing to advances in practice. These case studies will help to build up a clearer picture, but so far they point to a disconnect between mathematical theory and practice, which does not support Wigner’s conjecture during the long 19th century.