EXTREME VALUE THEORY AND LAWS OF RARE EVENTS: ABSTRACTS

Freddy Bouchet: Computation of rare events in Quasi-Geostrophic turbulent dynamics and the Allen Cahn equation.

Abstract: We will discuss several recent results for the computation of extremelly rare events for stochastic partial differential equations. The example of the stochastic two-dimensional Euler and Navier-Stokes equations, and quasi-geostrophic models, that describe planetary atmospheres will first be considered. We will also discussed at length the sampling of rare transitions in the Allen-Cahn equation. Two kinds of numerical algorithms will be discussed: action minimization methods that describe the most probable dynamical paths in dynamics forced by weak noises, and multilevel splitting algorithms that are adapted to a wide range of dynamical systems. The possible use of these algorithms for the computation of extreme statistics will be outlined. This is joint work with Jason Laurie, Joran Rolland, Eric Simonnet, and Jeroen Wouters.

Henk Bruin: Return time statistics: varying limits.

Abstract: The statistics of first returns to decreasing sets (holes U_r) can be viewed in several ways. One can scale time t as function of the size r of the hole as suggested by Kac Lemma. This frequently leads to exponential statistics. One can also let time go to infinity first, and obtain the so-called escape rate, and study this rate as the size of the hole tends to zero. Both approaches can be seen as special cases of a single scheme in which $r \to 0$, $t \to \infty$ along varying paths. In this talk I want to present some results in this direction, in connection with inducing techniques for interval maps. This is joint work in progress with Mark Demers (Fairfield) and Mike Todd (St. Andrews).

Juan-Juan Cai: Estimation of the marginal expected shortfall: the mean when a related variable is extreme

Abstract. Denote the loss return on the equity of a financial institution as X and that of the entire market as Y . For a given very small value of $p > 0$, the marginal expected shortfall (MES) is defined as $E(X|Y > Q_Y(1-p))$,

Date: July 4, 2014.

where $Q_Y(1-p)$ is the $(1-p)$-th quantile of the distribution of Y . The MES is an important factor when measuring the systemic risk of financial institutions. For a wide nonparametric class of bivariate distributions, we construct an estimator of the MES and establish the asymptotic normality of the estimator when $p \searrow 0$, as the sample size $n \to \infty$. Since we are in particular interested in the case $p = O(1/n)$, we use extreme value techniques for deriving the estimator and its asymptotic behavior. The finite sample performance of the estimator and the adequacy of the limit theorem are shown in a detailed simulation study. We also apply our method to estimate the MES of three large U.S. investment banks. This is joint work with John H.J. Einmahl, Laurens de Haan and Chen Zhou.

Key words and phrases. Asymptotic normality, conditional tail expectation, extreme values.

Davide Faranda: A recurrence-based technique for detecting genuine extremes in instrumental records

Abstract: We analyze several instrumental time series by using techniques originally developed for the analysis of extreme values of dynamical systems. We show that they have the same recurrence time statistics as a chaotic dynamical system perturbed with dynamical noise and by instrument errors. The technique provides a criterion to discriminate whether the recurrence of a certain event belongs to the normal variability or can be considered as a genuine extreme event with respect to a specific timescale fixed as parameter. The method gives a self-consistent estimation of the convergence of the statistics of recurrences toward the theoretical extreme value laws. We present examples on the analysis of temperature data and instrumental records of arterial and pulmonary blood pressure.

Stefano Galatolo: Logarithm laws, decay of correlations, skew products and arithmetical properties.

Abstract: We will consider the behavior of the time which is needed for a typical point to enter in a sequence of decreasing targets. This kind of problems can be equivalently reformulated as the time needed to reach an extremum of an observable along an orbit.

In several systems this time increases (having the same scaling behavior) as the inverse of the measure of the targets. We will see that a general condition for this to happen is superpolynomial decay of correlations. We will also see some applications, on Lorenz like flows and geodesic flows in variable negative curvature.

On the other hand there are systems, having particular arithmetical properties where the time needed to enter in a given sequence of balls increases much faster than the inverse of the ball's measure. We will see how this also happen in mixing systems, as skew products and reparametrizations of suitable flows.

Armelle Guillou: Robust and bias-corrected estimation of extreme failure sets

Abstract: In multivariate extreme value statistics, the estimation of probabilities of extreme failure sets is an important problem, with practical relevance for applications in several scientific disciplines. Some estimators have been introduced in the literature, though so far the typical bias issues that arise in application of extreme value methods and the non-robustness of such methods with respect to outliers were not addressed. We introduce a bias-corrected and robust estimator for small tail probabilities. The estimator is obtained from a second order model that is fitted to properly transformed bivariate observations by means of the minimum density power divergence technique. The asymptotic properties are derived under some mild regularity conditions and the finite sample performance is evaluated by a small simulation experiment. We illustrate the practical applicability of the method on a dataset from the actuarial context. (Coauthors: Christophe Dutang, Yuri Goegebeur.)

Mark Holland: Speed of convergence to an extreme value distribution

Abstract: We consider an ergodic dynamical system together with an observation function having a unique maximum at a (generic) point in the phase space. Under this observable, we consider the time series of successive maxima along typical orbits. Recent works have focused on the distributional convergence of such maxima (under suitable normalization) to an extreme value distribution. For certain dynamical systems, we establish convergence rates to the limiting distribution. In contrast to the case of i.i.d random variables, the convergence rates depend on the rate of mixing and the recurrence time statistics. For a range of applications, including uniformly expanding maps, quadratic maps, and intermittent maps, we establish corresponding convergence rates. We also establish convergence rates for certain hyperbolic systems such Anosov systems, and discuss convergence rates for non-uniformly hyperbolic systems, such as Hénon maps. Work is joint with M. Nicol.

Yuri Kifer: Poisson and compound Poisson asymptotics in conventional and nonconventional setups

Abstract: The Poisson limit theorem which appeared in 1837 seems to be the first law of rare events in probability. Various generalizations of it and estimates of errors of Poisson approximations were obtained in probability and more recently this became a popular topic in dynamics in the form of study of asymptotics of numbers of arrivals at small (shrinking) sets by a stochastic process or by a dynamical system. I will describe recent results on Poisson and compound Poisson asymptotics in a nonconventional setup, i.e. for numbers of events of multiple returns to shrinking sets, namely, for numbers of combined events of the type $\{\omega : \xi(jn, \omega) \in \Gamma_N, j = 1, \ldots, \ell\}$, $n \leqslant N$ where $\xi(k, \omega)$ is defined as a stochastic process from the beginning or it is built from a dynamical system by writing $\xi(k, \omega) = T^k\omega$. We obtain an essentially complete description of possible limiting behaviors of distributions of numbers of multiple recurrencies to shrinking cylinders for ψ-mixing shifts. Some possible extensions and related questions will be discussed, as well. Most of the results were obtained jointly with my student Ariel Rapaport and some of them are new even for the widely studied single (conventional) recurrencies case.

Natalia Markovich: Inferences for clusters of extreme values: modeling, distributions, applications

Abstract: Extremes in stochastic sequences arising as clusters of exceedances over a threshold are observed in numerous applications of climate research, finance, telecommunication and social systems. Such clusters are caused by dependence. We define the cluster as a conglomerate containing consecutive exceedances of the underlying process $\{R_n\}$ over a threshold u separated by return intervals with consecutive non-exceedances. The geometric-like limit distributions of the cluster size and inter-cluster size are obtained in Markovich (2014) under the mixing condition similar to one used in Ferro and Segers (2003). The sequence of high $(1 - \rho_n)$-order quantiles $x_{\rho n}$ of the $\{R_n\}$ is used as thresholds. The obtained distributions differ from the geometric distribution by the extremal index of the process $\{R_n\}$. The asymptotic first moments of both cluster characteristics are obtained Markovich (2013), Markovich (2014). Similarly, the result can be extended to moments of higher orders. In Markovich (2014) the duration of (inter-)clusters is defined as a sum of the random number of weakly dependent, regularly varying inter-arrival times between events of interest with tail index $0 < \alpha < 2$. Under anti-clustering conditions proposed in Basrak et al. (2010) the tail of limit distributions arising from the cluster and inter-cluster durations are derived to be bounded by the tail of a stable distribution. The exposition is accompanied with examples motivated by applications.

References:

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Thomas Mikosch: Asymptotic theory for the sample covariance matrix of a heavy-tailed multivariate time series

Abstract: We give an asymptotic theory for the eigenvalues of the sample covariance matrix of a multivariate time series. The time series constitutes a linear process across time and between components. The input noise of the linear process has regularly varying tails with index $\alpha \in (0, 4)$; in particular, the time series has infinite fourth moment. We derive the limiting behavior for the largest eigenvalues of the sample covariance matrix and show point process convergence of the normalized eigenvalues. The limiting process has an explicit form involving points of a Poisson process and eigenvalues of a non-negative definite matrix. Based on this convergence we derive limit theory for a host of other continuous functionals of the eigenvalues, including the joint convergence of the largest eigenvalues, the joint convergence of the largest eigenvalue and the trace of the sample covariance matrix, and the ratio of the largest eigenvalue to their sum. This is joint work with Richard A. Davis (Columbia NY) and Oliver Pfaffel (Munich).

Philippe Naveau: Statistical analysis of heavy rainfall in France via multivariate extreme value theory

Abstract: Analysing heavy rainfall in France is complex due to the high number of weather stations and the complexity of weather system patterns over the French territory. This leads to computational issues and classical Extreme Value Theory (ETV) cannot be directly applied. To bypass the computational hurdles, we perform a dimension reduction approach, based on EVT concepts, to create independent regional clusters. Within each

cluster, we propose a nonparametric approach for estimating the maxima dependence function. This is a joint work with S. A. Padoan, G. Marcon, E. Bernard, M. Vrac, O. Mestre.

Holger Rootzen: Extreme values of stochastic processes

Abstract: This talk will provide a quick and incomplete list of classical/old results on extremes and crossings of stochastic processes and fields, intended for dynamical systems researchers who may not be completely familiar with the area. More recent results will be provided by other speakers in this workshop.

Jérôme Rousseau: Hitting time statistics for random dynamical systems

Abstract: We study law of rare events for random dynamical systems. We obtain an exponential law (with respect to the invariant measure of the skew-product) for super-polynomially mixing random dynamical systems. For random subshifts of finite type, we analyze the distribution of hitting times with respect to the sample measures. We prove that with a super-polynomial decay of correlations one can get an exponential law for almost every point and with stronger mixing assumptions one can get a law of rare events depending on the extremal index for every point. (These are joint works with Benoit Saussol and Paulo Varandas, and Mike Todd).

Benoît Saussol: Quenched hitting time distribution in random subshifts

Abstract: We study the distribution of hitting times for a class of random dynamical systems. We prove that for invariant measures with super-polynomial decay of correlations hitting times to dynamically defined cylinders satisfy exponential distribution. Similar results are obtained for random expanding maps. We emphasize that what we establish is a quenched exponential law for hitting times. (This is joint work with Jrme Rousseau and Paulo Varandas).

Johan Segers: An M-estimator of spatial tail dependence

Abstract: Tail dependence models for distributions attracted to a max-stable law are fitted using observations above a high threshold. To cope with spatial, high-dimensional data, a rank-based M-estimator is proposed relying on bivariate margins only. A data-driven weight matrix is used to minimize the asymptotic variance. Empirical process arguments show that

the estimator is consistent and asymptotically normal. Its finite-sample performance is assessed in simulation experiments involving popular max-stable processes perturbed with additive noise. An analysis of wind speed data from the Netherlands illustrates the method.